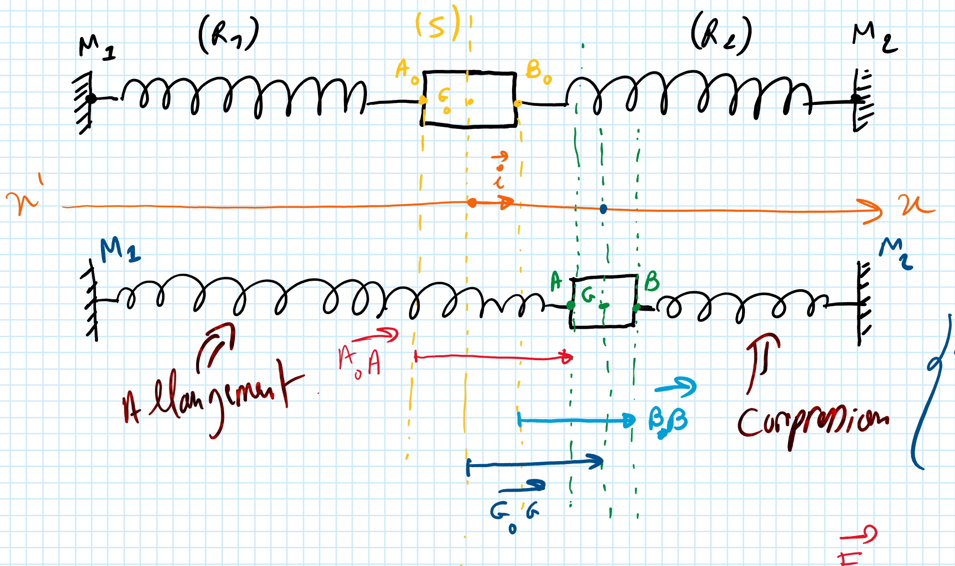
BeInBac.com
+212 719-411876

Oscillateur avec deux Ressorts

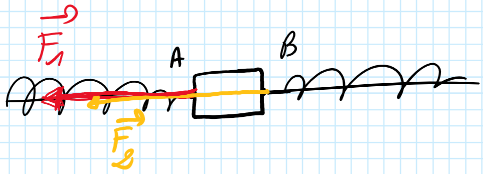
I



$K = K_1 = K_2$

$$\vec{G_0G} = x\,\vec{i} = \vec{A_0A} = \vec{B_0B}$$

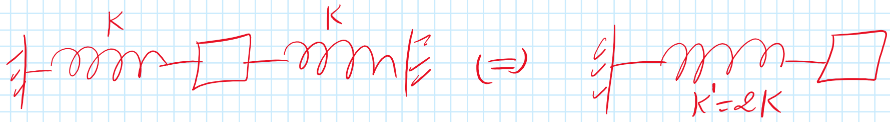
$$\begin{cases} \vec{F_1} = -K_1\,\vec{A_0A} = -K_1\,x\,\vec{i} \\ \vec{F_2} = -K_2\,\vec{B_0B} = -K_2\,x\,\vec{i} \end{cases}$$



$$\boxed{K = K_1 = K_2} \Rightarrow \vec{F_1} = \vec{F_2} = -K\,x\,\vec{i}$$

$$\vec{P} + \vec{R} + \vec{F_1} + \vec{F_2} = m\,\vec{a_G} \Rightarrow \text{proj}/_{Ox} : -Kx - Kx = m\ddot{x}$$

$$\Rightarrow \underbrace{\ddot{x} + \frac{2K}{m}\,x = 0}_{} \Rightarrow \ddot{x} + \frac{K'}{m}\,x = 0$$



$$\omega_0^2 = \frac{2K}{m} = \frac{K'}{m} = \frac{K_1 + K_2}{m}$$

$$T_0 = 2\pi\sqrt{\frac{m}{K'}} = 2\pi\sqrt{\frac{m}{2K}} = 2\pi\sqrt{\frac{m}{K_1 + K_2}}$$

- $E_c = \frac{1}{2} m \dot{x}^2$; $E_{Pe} = E_{Pe_1} + E_{Pe_2} = \frac{1}{2}K x^2 + \frac{1}{2} K x^2 + cte$
$= K x^2 + cte$

$$E_m = \frac{1}{2} m \dot{x}^2 + K x^2 + cte = \text{Constante}$$

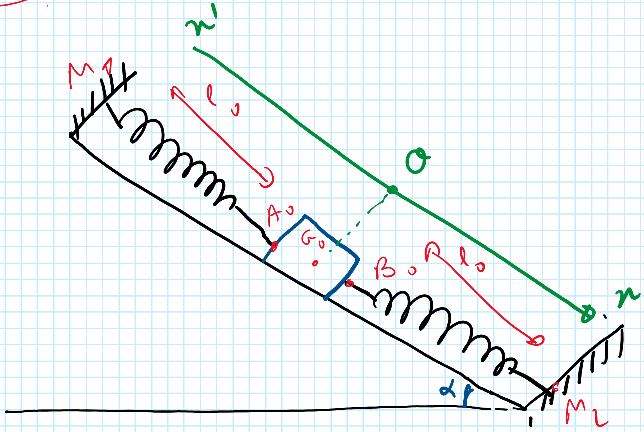
$$\frac{dE_m}{dt} = 0 \Rightarrow m\dot{x}\ddot{x} + 2kx\dot{x} = 0$$

$$\underset{\neq 0}{\dot{x}}\big(\underbrace{m\ddot{x} + 2kx}_{0}\big) = 0$$

$$\boxed{\ddot{x} + \frac{2k}{m}x = 0}$$

III

On incline l'ensemble d'un angle α / horizontal



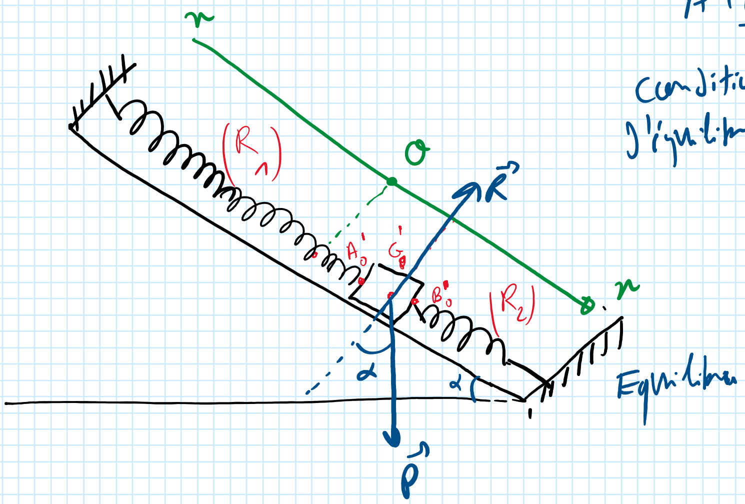
$$\vec{A_0A_0'} = \vec{B_0B_0'} = \vec{G_0G_0'} = x_0\vec{i} = \Delta l_0\vec{i}$$

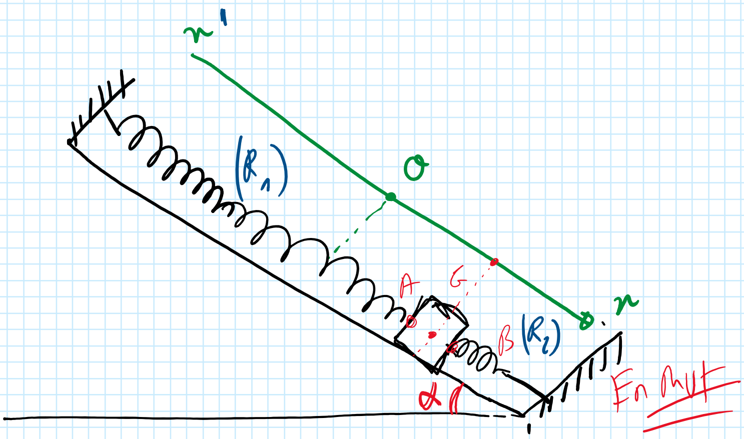
$$\left.\begin{aligned} \vec{F}_{10} &= -k_1\vec{A_0A_0'} = -kx_0\vec{i} \\ \vec{F}_{20} &= -k_2\vec{B_0B_0'} = -kx_0\vec{i} \end{aligned}\right\} \vec{F}_{10} = \vec{F}_{20}$$

À l'équilibre : $\vec{P} + \vec{R} + \vec{F}_{10} + \vec{F}_{20} = \vec{0}$

Condition d'équilibre → $\boxed{mg\sin\alpha - 2kx_0 = 0}$

$$x_0 = \Delta l_0 = \frac{mg\sin(\alpha)}{2k}$$





$$\vec{A_0A} = \vec{B_0B} = \vec{G_0G} = (x_0 + x)\vec{i}$$

$$\left\{\begin{aligned} \vec{F}_1 &= -k_1\vec{A_0A} = -k(x_0 + x)\vec{i} \\ \vec{F}_2 &= -k_2\vec{B_0B} = -k(x_0 + x)\vec{i} \end{aligned}\right.$$

$$\vec{P} + \vec{R} + \vec{F}_1 + \vec{F}_2 = m\vec{a}$$

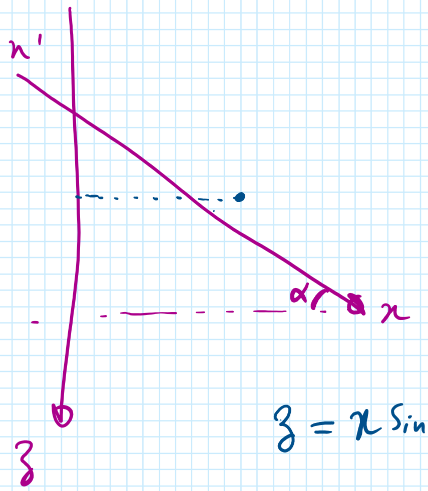
$$mg\sin\alpha + 0 - 2k(x_0 + x) = m\ddot{x}$$

$$\Rightarrow \underbrace{mg\sin\alpha - 2kx_0}_{0} - 2kx = m\ddot{x}$$

$$-2kx = m\ddot{x} \Rightarrow \ddot{x} + \frac{2k}{m}x = 0$$

$$\ddot{x} + \frac{k'}{m}x = 0 \quad \text{avec } k' = 2k$$

$$\omega_0^2 = \frac{2k}{m} = \frac{k_1 + k_2}{m}$$



- $E_{pp} = -mgz + c_1 = -mgx\sin\alpha + c_2$
- $E_{pe} = E_{pe1} + E_{pe2}$

$z = x\sin\alpha$

$$\vec{A_0A} = \vec{B_0B} = \vec{G_0G} = (x_0 + x)\vec{i}$$

$$\vec{F_1} = \vec{F_2}$$

$$\Delta\ell_1 > 0 \text{ et } \Delta\ell_2 < 0 \Rightarrow \underbrace{\Delta\ell_1 \vec{i}}_{\vec{F_1}} = \underbrace{-\Delta\ell_2 \vec{i}}_{\vec{F_2}} \Rightarrow \Delta\ell_1 = -\Delta\ell_2 = x_0 + x$$

$$\Rightarrow \Delta\ell_1 = x_0 + x \quad \text{et} \quad \Delta\ell_2 = -x_0 - x = -(x_0 + x)$$

$= \ell_{f2} - \ell_0$

$\ell_{f1} - \ell_0$

$$E_{pe1} = \frac{1}{2}k(\Delta\ell_1)^2 + c_2 = \frac{1}{2}k(x_0 + x)^2 + c_2$$

$$E_{pe2} = \frac{1}{2}k(\Delta\ell_2)^2 + c_3 = \frac{1}{2}k(x_0 + x)^2 + c_3$$

$$E_p = -mgx\sin\alpha + k(x_0 + x)^2 + cte$$

$$E = \frac{1}{2}m\dot{x}^2 - mgx\sin\alpha + k(x_0 + x)^2 + cte \equiv \text{Constante}$$

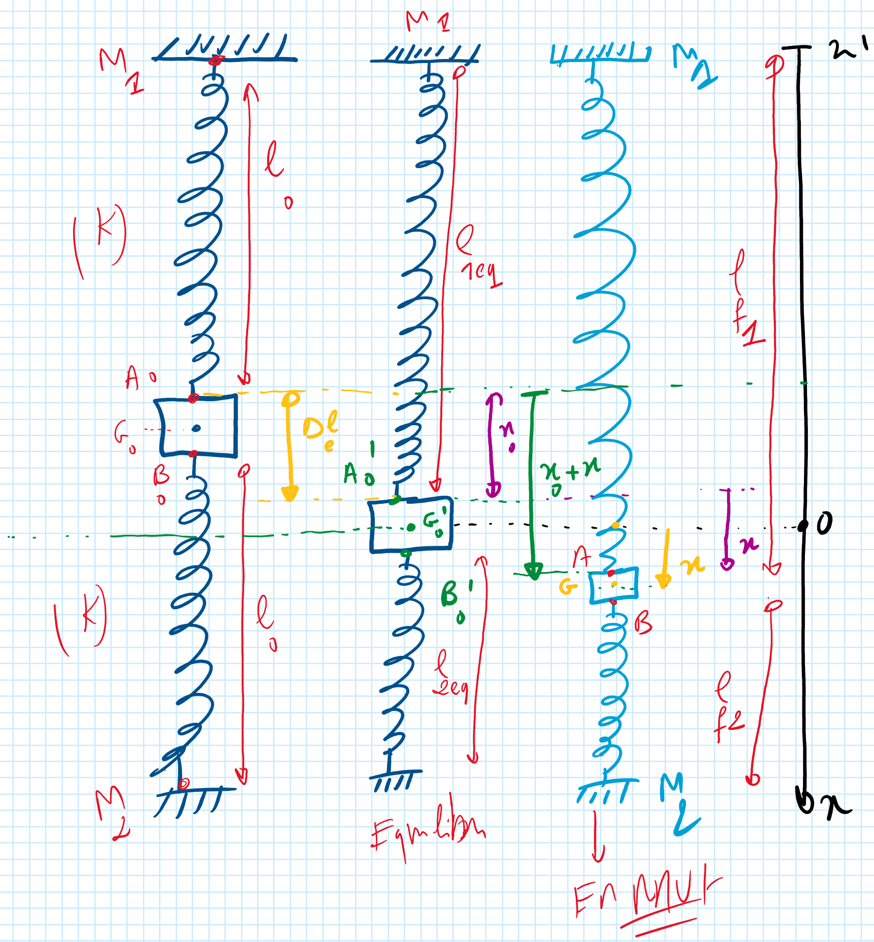
$$E_m = \frac{1}{2} m \dot{x}^2 - m g x \sin\alpha + k(x_0 + x)^2 + cte = \text{Constante}$$

$$\frac{dE_m}{dt} = 0 \Rightarrow m\cancel{\dot{x}}\ddot{x} - mg\cancel{\dot{x}}\sin\alpha + 2k(x_0 + x)\cancel{\dot{x}} = 0$$

$$m\ddot{x} \underbrace{- mg\sin\alpha + 2kx_0}_{0} + 2kx = 0$$

$$\boxed{\ddot{x} + \frac{2k}{m}x = 0}$$

III



$$\vec{G_0G_0'} = \vec{A_0A_0'} = \vec{B_0B_0'} = \Delta\ell_e \vec{i}$$

$$\vec{F}_{10} = -k_1 \vec{A_0A_0'} = -k\,\Delta\ell_e \vec{i}$$

$$\vec{F}_{20} = -k_2 \vec{B_0B_0'} = -k\,\Delta\ell_e \vec{i}$$

$$\vec{P} + \vec{F}_{10} + \vec{F}_{20} = \vec{0}$$

$$mg - k\Delta\ell_e - k\Delta\ell_e = 0$$

$$mg = 2k\Delta\ell_e = 2kx_0$$

$$\boxed{\Delta\ell_e = \frac{mg}{2k}} = x_0 > 0$$

$$\Delta\ell_{1eq} = \ell_{1eq} - \ell_0 > 0$$

$$\Delta\ell_{2eq} = \ell_{2eq} - \ell_0 < 0$$

$$\begin{cases} \Delta\ell_{1eq} = \ell_{1eq} - \ell_0 = \Delta\ell_e = x_0 = mg/2k \\ \Delta\ell_{2eq} = \ell_{2eq} - \ell_0 = -\Delta\ell_e = -x_0 = -mg/2k \end{cases}$$

$$= (\ell \;\; - \ell_0) - (\ell \;\; - \ell_0)$$

$$\ell_{1eq} - \ell_{2eq} = \ell_{1eq} - \ell_0 + \ell_0 - \ell_{2eq} = \underbrace{(\ell_{1eq} - \ell_0)}_{x_0} - \underbrace{(\ell_{2eq} - \ell_0)}_{-x_0}$$

$$= 2x_0 = 2\Delta\ell_e = 2 \cdot \frac{mg}{2K} = \frac{mg}{K}$$

En Mvt

$$\overrightarrow{A_0A} = \overrightarrow{B_0B} = \overrightarrow{G_0G} = (\Delta\ell_e + x)\vec{i} = (x_0 + x)\vec{i}$$

$$\vec{F_1} = -K_1 \overrightarrow{A_0A} = -K(x_0 + x)\vec{i}$$

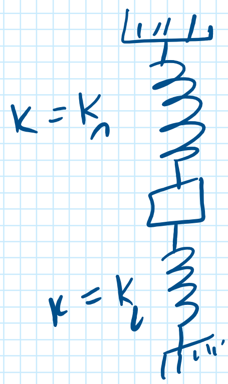
$$\vec{F_2} = -K_2 \overrightarrow{B_0B} = -K(x_0 + x)\vec{i}$$

PFD

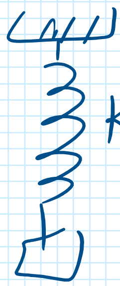
$$\vec{P} + \vec{F_1} + \vec{F_2} = m\vec{a} \Rightarrow mg - 2K(x_0 + x) = m\ddot{x}$$

$$\underbrace{mg - 2Kx_0}_{0} - 2Kx = m\ddot{x}$$

$$\Rightarrow \ddot{x} + \frac{2K}{m}x = 0 \iff \ddot{x} + \frac{K'}{m}x = 0$$



$\Leftrightarrow$



$K' = K_1 + K_2 = 2K$

$\overrightarrow{A_0A} = \overrightarrow{B_0B}$

$$\Rightarrow \vec{F_1} = \vec{F_2} \quad \Delta\ell_1 \vec{i} = -\Delta\ell_2 \vec{i}$$

$\ell_{f1} - \ell_0 = \Delta\ell_1 > 0$

$\ell_{f2} - \ell_0 = \Delta\ell_2 < 0$

$$\begin{cases} \Delta\ell_1 = \Delta\ell_e + x = x_0 + x \\ \Delta\ell_2 = -(\Delta\ell_e + x) = -(x_0 + x) \end{cases}$$

$$\begin{cases} E_{pe} = E_{pe1} + E_{pe2} \\ = \frac{1}{2}K(x_0+x)^2 + \frac{1}{2}K(x_0+x)^2 + ct_1 \end{cases}$$

- $E_{pe} = K(x_0+x)^2 + ct_1$
- $E_{pp} = -mgx + ct_2$

$$E_m = \frac{1}{2}m\dot{x}^2 - mgx + K(x_0+x)^2 + ct = \text{Constante}$$

$$\frac{dE_m}{dt} = 0 \Rightarrow \boxed{\begin{array}{l} \ddot{x} + \frac{2K}{m}x = 0 \\ \ddot{x} + \frac{K_1+K_2}{m}x = 0 \end{array}}$$