Séance 8 : Les EDLs second ordre

dimanche 15 mai 2022 16:27

1) $y'' + y' - 6y = 2e^{3x}$	2) $y'' - 4y' + 3y = 6x + 1 + 4e^{x} + 8e^{-x}$
3) $y'' - 2y' + y = (x^2+1)e^{x}$	4) $y'' - 4y' + 13y = 10\cos 2x + 25\sin 2x$
5) $y'' + y = \cos^3 x$	6) $y'' - 2y' + 5y = \cosh x \cos 2x$
7) $y'' - 2y' + 2y = e^{x}\sin x$	8) $y'' - 4y' + 13y = e^{-2x}\cos x$

$k_2)\ \sin(2x)\, e^{x}$ $\frac{+e^{x}}{2}$

1. E. SSM : $g(x) = 0$:

$$y'' + y' - 6y = 0$$

$$r^2 + r - 6 = 0$$

$$(r-2)(r+3) = 0$$

$$r_1 = 2, \quad r_2 = -3$$

$$y(x) = \lambda e^{2x} + \mu e^{-3x}, \quad \lambda, \mu \in \mathbb{R}.$$

ii) " $y'' + y - 6y = 2e^{3x}$ " $= P(x) \cdot e^{3x}$ $\qquad P(x) = 2 \cdot x^0 \qquad \deg(P) = 0.$

le second membre est une expo : e^{kx}, $k \notin \{r_1, r_2\}$.

la forme des solutions : $y_p = k \cdot e^{3x}$ $\qquad \deg(Q) = \deg(P) = 0$

$$y_p'' + y_p' - 6y_p = 2 \cdot e^{3x}$$

$$9k \cdot \cancel{e^{3x}} + 3k\,\cancel{e^{3x}} - 6k\cancel{e^{3x}} = 2 \cdot \cancel{e^{3x}}$$

$$6k = 2 \Rightarrow k = 1/3$$

$$y_p = \frac{1}{3} \cdot e^{3x}$$

iii) $y_G = y_p + y_{SSM}$

$$y_G = \frac{1}{3} e^{3x} + \lambda e^{2x} + \mu \cdot e^{-3x} \qquad (\lambda, \mu \in \mathbb{R})$$

2) $y'' - 4y' + 3y = 6x + 1 + 4e^{x} + 8e^{-x}$

i) ESSM :

$$y'' - 4y' + 3y = 0$$

$$r^2 - 4r + 3 = 0$$

$$(r-1)(r-3) = 0$$

$$y_{SSM} = \lambda e^{1x} + \mu \cdot e^{3x} \qquad (\lambda, \mu \in \mathbb{R}.)$$

ii) le second membre $g(x) = 6x + 1 + 4e^{x} + 8e^{-x}$ est une somme de :

- un poly. du 1er degré : $6x + 1$
- une exp. $e^{k_1 x}$, $k_1 = 1 \in \{r_1, r_2\} = \{1, 3\}$
- une exp : $e^{k_2 x}$, $k_2 = -1 \notin \{r_1, r_2\} = \{1, 3\}$.

Nous allons chercher une solution particulière qui soit la somme de :

a) + $y'' - 4y' + 3y = 6x + 1$

b) + $y'' - 4y' + 3y = 4 \cdot e^{x}$

a) + $y'' - 4y' + 3y = 6x+1$

b) + $y'' - 4y' + 3y = 4 \cdot e^{x}$

c) + $y'' - 4y' + 3y = 8e^{-x}$.

a) $y'' - 4y' + 3y = 6x+1$.

$y_{p_1} = ax+b$

$y'_{p_1} = a$ $\quad\quad 0 - 4a + 3ax + 3b = 6x+1$

$y''_{p} = 0$ $\quad\quad \begin{cases} 3a = 6. \Rightarrow a = 2 \\ 3b - 4a = 1 \Rightarrow b = 3. \end{cases}$

$y_{p_1} = 2x+3.$

b) $y'' - 4y' + 3y = 4e^{1x}$.

$y_{p_2} = a.x.e^{x}$. $\quad\quad a = -2$ $\quad y_{p_2} = -2.x e^{x}$.

$y''_{p} = a(x+2)e^{x}$

$y'_{p} = a(x+1)e^{x}$

c) $y'' - 4y' + 3y = 8e^{-x}$

$y_{p_3} = a.e^{-x}$

$y'_{p_3} = -a \cdot e^{-x}$ $\quad\quad a\cancel{e^{-x}} + 4.a\cancel{e^{-x}} + 3.a\cancel{e^{-x}} = 8\cancel{e^{-x}}$

$8a = 8 \Rightarrow a = 1$

$y''_{p_3} = +ae^{-x}$ $\quad\quad y_{p_3} = e^{-x}$.

SPEASM : $\quad y = y_{p_1} + y_{p_2} + y_{p_3} = 2x+3 - 2xe^{+x} + e^{-x}$

iii) **SG :** $y_G = \lambda e^{x} + \mu e^{-3x} + 2x+3 - 2xe^{+x} + e^{-x}$ $\quad (\lambda, \mu \in \mathbb{R})$

3°) $y'' - 2y' + y = (x^2+1)e^{x} = P(x)e^{x}$ $\quad\quad$ deg P = deg Q = 2.

i) SSM: $\quad y'' - 2y' + y = 0.$

$r^2 - 2r + 1 = 0$

$(r-1)^2 = 0$

"1" est une racine double.

$y = (ax+b)e^{1x}$ $\quad\quad (a, b \in \mathbb{R})$

ii) SEASM:

$g(x) = P(x)e^{\alpha x}$ $\quad\quad \alpha = 1$

$$y_p = e^x \cdot x^2 \cdot Q(x)$$

$Q(x) = ax^2 + bx + c.$

$$y_p = e^x(ax^4 + bx^3 + cx^2).$$

$$y_p' = e^x(ax^4 + bx^3 + cx^2) + e^x(4ax^3 + 3bx^2 + 2cx).$$

$$= e^x(ax^4 + (b+4a)x^3 + (c+3b)x^2 + 2cx)$$

$$y_p'' = e^x(a - \quad - \quad -) + e^x(4ax^3 + 3(b+4a)x^2$$

$$+ 2(c+3b)x + 2c)$$

$a = \frac{1}{12}$, $b = 0$, $c = 1/2$

$$y_p = \left(\frac{x^4}{12} + \frac{x^2}{2}\right) \cdot e^x$$

iii) $y_G = y_p + y_{SSH} = \left(\frac{x^4}{12} + \frac{1}{2}x^2\right)e^x + (ax+b)e^x$, $a, b \in \mathbb{R}$.

4°) $y'' - 4y' + 13y = (10\cos(2x) + 25\sin(2x))e^{0x}$ $\qquad$ $0 + 2i$

i) $y'' - 4y' + 13y = 0.$

$r^2 - 4r + 13 = 0$

$\Delta = 16 - 4 \cdot 13 = 16 - \quad r^2 = (6i)^2$

$$\begin{cases} r_1 = \frac{4 - 6i}{2} = 2 - 3i \\ r_2 = \bar{r}_1 = 2 + 3i \end{cases}$$

$$y_{SSM} = (k_1 \cos 3x + k_2 \sin(3x))e^{2x} \quad k_1, k_2 \in \mathbb{R}.$$

ii) SASM:

$$y_p = a\cos(2x) + b\sin(2x).$$

y_p'

y_p'' $\qquad$ $\begin{cases} a = 2 \\ b = 1 \end{cases}$

$$y_p = 2\cos(2x) + \sin(2x)$$

iii) $y_G = 2\cos(2x) + \sin(2x) + (k_1\cos(3x) + k_2\sin(3x))e^{2x}$ $(k_1, k_2 \in \mathbb{R})$

5) $y'' + y = \cos^3(x):$

$$\cos^3(x) = \frac{1}{8}\left(e^{ix}+e^{-ix}\right)^3 = \dots = \frac{1}{4}\cdot\left(\cos(3x)+\cos(x)\right).$$

$$y''+y = \frac{1}{4}\cdot\cos(3x) + \frac{1}{4}\cdot\cos(x)$$

SSM:

$$r^2+1=0$$

$$r=\pm i$$

$$y_{SSM} = k_1\cos(x) + k_2\cdot\sin(x).$$

ii) $\begin{cases} y''+y = \frac{1}{4}\cdot\cos(3x) \rightsquigarrow y_p = -\frac{1}{32}\cos(3x) \\ y''+y = \frac{1}{4}\cdot\cos(x) \rightsquigarrow y_p = \frac{1}{8}x\cdot\sin(x). \end{cases}$

$$y_p = y_{p_1}+y_{p_2} = -\frac{1}{32}\cos(3x)+\frac{1}{8}\cdot x\cdot\sin(x).$$

iii) $y_G = -\frac{1}{32}\cos(3x) + \frac{1}{8}\cdot x\cdot\sin(x) + k_1\cos(x) + k_2\sin(x).$

7) $y''-2y'+2y = e^x\cdot\sin(x)+$ $\quad \alpha+i\beta = 1+i$

i) SSM: $\quad r^2-2r+2=0.$

$$(r-1-i)(r-1+i)=0 \quad r_1 = 1+i,\ r_2 = 1-i$$

$$y_{SSM} = \left(k_1\cos(x)+k_2\cdot\sin(x)\right)e^x$$

ii) SASM: $\quad y_p = x\cdot e^x\left(a\cos(x)+b\sin(x)\right)$

$$b=0 \qquad a=-1/2$$

$$y_p = -\frac{1}{2}x\cos(x)$$

iii) $y_G = -\frac{1}{2}x\cdot\cos(x) + \left(k_1\cos(x)+k_2\sin(x)\right)e^x \quad (k_1, k_2\in\mathbb{R})$